

SYNOPSIS

A STUDY OF SOME CHARACTERISTICS OF BLOOD FLOW WITH SPECIAL REFERENCE TO CARDIOVASCULAR DISEASES



**Submitted for the Degree of
DOCTOR OF PHILOSOPHY
IN
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PURPOSE OF STUDY:

Mathematical and theoretical biology is an interdisciplinary scientific research field with a range of applications in biology, medicine and biotechnology. The field may be referred to as mathematical biology or biomathematics to stress the mathematical side, or as theoretical biology to stress the biological side. It includes at least four major subfields: biological mathematical modeling, relational biology/complex systems biology (CSB), bioinformatics and computational biomodeling/ biocomputing. Mathematical biology aims at the mathematical representation, treatment and modeling of biological processes, using a variety of applied mathematical techniques and tools. It has both theoretical and practical applications in biological, biomedical and biotechnology research. For example, in cell biology, protein interactions are often represented as "cartoon" models, which, although easy to visualize, do not accurately describe the systems studied. In order to do this, precise mathematical models are required. By describing the systems in a quantitative manner, their behavior can be better simulated, and hence properties can be predicted that might not be evident to the experimenter.

Such mathematical areas as calculus, probability theory, statistics, linear algebra, abstract algebra, graph theory, combinatorics, algebraic geometry, topology, dynamical systems, equations and coding theory are now being applied in biology. Some mathematical areas, such as statistics, were developed as tools during the conduct of research into mathematical biology.

Blood flow is the continuous running of blood in the cardiovascular system. The human body is made up of several processes all carrying out various functions. We have the gastrointestinal system which aids the digestion and the absorption of food. We also have the respiratory system which is responsible for the absorption of O₂ and elimination

of CO_2 . The urinary system removes waste from the body. The cardiovascular system helps to distribute food, O_2 and other product of metabolism. The reproductive system is responsible for perpetuating the species. The nervous and endocrine system is responsible for coordinating the integration and function of other system.

The term stenosis denotes the narrowing the artery due to development of arteriosclerotic plaques or other types of abnormal tissue development. As the growth projects in the cavity of artery, blood flow is obstructed. The obstruction may damage the internal cells of the wall and may lead to further growth of stenosis. Thus, there is a coupling between the growth of a stenosis and the flow of blood in artery since they affect each other. Many arterial diseases such as high cholesterol, diabetes etc. are caused by obstruction or stenosis in arteries. Our main purpose is to study the application of some mathematical model of blood flow through Renal Artery stenosis (RAS), pulmonary stenosis, Aortic stenosis etc.

The heart together with all the blood vessels makes up the cardiovascular system. This system is responsible for supplying oxygen to all the tissues in the body as well as removing waste products. The diseases related with the heart and the blood vessels are known as cardiovascular diseases. Atherosclerosis is a kind of cardiovascular disease. It is a condition in which an artery wall thickens as a result of the accumulation of macrophage white blood cells and promoted by low-density lipoproteins (plasma proteins that carry cholesterol and triglycerides). It is a chronic disease that remains asymptomatic for decades.

The aim of our research is to solve local and global blood flow problems through stenosed arteries and provide more information for the further investigations to scientists and

biologists. We are also interested to extend some problems and modeling of some new problems related to our research field. We are also paying attention to solve these mathematical models.

Numerical and graphical analysis will be performed for the validity of the results. Our study would be at par with the investigations performed in India and abroad.

The intention of our work is to provide more information about human blood flow under stenotic condition to cure the patients suffering from cardiovascular diseases. We would also face the modeling related to my research theme. We are also interested to solve these mathematical models numerically and graphically by using MATLAB 7.8.

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(A) **Title: FLOW CHARACTERISTICS OF BLOOD FLOW UNDER STENOTIC CONDITIONS WITH SPECIAL REFERENCE TO CARDIOVASCULAR DISEASES**

(B) **PRESENT STATE OF KNOWLEDGE:**

In the past there have been a large number of investigations to examine blood flow in the cardiovascular system. The study has created a lot of interest in such type of blood flow. As such sound literature in this area of blood flow is available. Various investigators have studied Blood flows through stenosed arteries. We do not have it mind to provide the complete historical review. However, following are the significant human resources responsible for the vast development of literature in this trend.

Ku (1997) investigated that the blood flow in arteries is dominated by unsteady flow phenomena. He discussed that a non dimensional frequency parameter, the Womersley number, governs the relationship between the unsteady and viscous forces. Atherosclerotic disease tends to be localized in these sites and results in a narrowing of the artery lumen—a stenosis. **Usha and Prema (1999)** applied a particle-fluid suspension model to the problem of pulsatile blood flow through a circular tube under the influence of body acceleration. The analytic expressions for axial velocity for both, fluid and particle phase, fluid acceleration, wall shear stress and instantaneous flow rate have been obtained. Assuming flow of blood to be Newtonian, unsteady and viscous, **Sanyal and Maji (1999)** studied the oscillatory flow characteristics of blood in a single constricted blood vessel taking an indented tube for mild stenosis. The effect of stenosis on pressure gradient is shown. **El-Shehawey et al. (2000)** studied the nature of unsteady flow of blood in the presence of magnetic field through a circular pipe taking blood as an electrically

conducting, visco-elastic, non-Newtonian fluid. Two –fluid model analyses have been carried out by **Srivastava (2000)** to observe the effects of a non-symmetrical stenosis on blood flow characteristics.

Sarma and Sut (2001) developed a numerical model to study the effect of magnetic field on pulsatile flow of blood in a porous channel. They observed that when the Hartmann number increases, the fluid velocity as well as magnitude of mass flux decreases. **Liu and Yamaguchi (2001)** found out a systematic study of a pulsatile flow in a stenosed channel to identify how the waveform affects the generation, development and breakdown of the vortex wave. **Srivastava (2002)** observed that the resistance to flow decreases with increasing shape parameter but increases with hematocrit (red cell concentration). **Stroud et al. (2002)** carried out a numerical analysis of flow through a severely stenotic carotid artery bifurcation. They considered both steady and pulsatile flow conditions for different Reynolds numbers. They found that both dynamic pressure and wall shear stress were very high, proximal to the stenosis throat in the internal carotid artery. They also observed vortex shedding downstream of the most severe occlusion.

Varghese and Frankel (2003) obtained numerical predictions for computational pulsatile flow through different axisymmetric stenosis within the frame work of two equation turbulence models. The effects of viscoelastic wall properties and micropolar fluid parameters on the flow are investigated by **Muthu et al. (2003)** using the equation of fluid as well as of the deformable boundaries. A perturbation technique is used to determine flow characteristic by them. The use of catheters is of immense importance in many areas of scientific significance and has become a criterion for diagnosis and treatment of cardiovascular diseases. **Liu et al. (2004)** established a mathematical model of a rigid tapered artery with axially symmetric stenosis.

Pralhad and Schultz (2004) computed the flow parameters such as velocity, resistance to flow, and shear stress distribution for different suspension concentrations (haematocrit), and for the blood diseases; polycythemia, plasma cell dyscrasias, and for Hb SS (sickle cell). **Dabiri et al. (2005)** constructed a software that can determine pressure and flow at different points of an arbitrary arterial networks. **Mekheimer (2005)** studied the peristaltic transport of a viscous incompressible fluid through the gap between coaxial tubes, where the outer tube is non-uniform and a sinusoidal wave travelling down its wall and the inner tube is rigid, uniform and moving with a constant velocity.

Rathod et al. (2006) have studied the pulsatile flow of blood through rigid inclined circular tubes under the influence of periodic body acceleration. The blood flow in human arteries has been analytically calculated according to Poiseuille's equation by **Gabry's et al. (2006)**. Assuming blood to be a couple stress fluids, **Sanyal et al. (2007)** investigated the effect of magnetic field on pulsatile motion of blood through an inclined circular tube with periodic body acceleration. **Kolachalama et al. (2007)** proposed a diagnostic technique to identify patients with carotid stenosis who could most benefit from angioplasty followed by stent implantation. **Bhardwaj and Kanodia (2007)** considered the pulsatile flow of blood through a stenosed porous medium under periodic body acceleration without considering magnetic effect. **Gentile et al. (2008)** analyzed the longitudinal transport of nanoparticles in blood vessels with blood described as Casson fluid. An explicit expression has been derived for the effective longitudinal diffusion depending non-linearly on the rheological parameter, the ratio between the plug and vessel radii and on the permeability parameters, related to the hydraulic conductivity and pressure drop across the vessel wall respectively.

Prasad and Radhakrishnaamacharya (2008) studied that flow resistance increases with the height of stenosis, yield stress, power law index but decreases with the inclination and shear stress increases with plug core region radius.

Layek et al. (2009) investigated the effects of an overlapping stenosis on flow characteristics considering the pressure variation in both the radial and axial directions of the arterial segment. **Kumar and Kumar (2009)** obtained pressure drop, flux and pressure gradient with rigid tapered grafts for Newtonian and non-Newtonian fluids. They found that flux, pressure drop and pressure gradient increase with the taper as the radius decreases for Newtonian fluid. **Shah (2010)** proposed a mathematical model to describe blood flow through an axially non-symmetric but radially symmetric stenosed artery when blood is represented as power-law fluid and a uniform magnetic field is applied on the flow. She observed that the magnitudes of the blood flow characteristics significantly increase with in the red cell concentration, which is depending on hematocrit value of blood. **Mishra et al (2010)** investigated the wall shear stress, resistance parameter and flow rate across mild stenosis situated symmetrically on steady blood flow through blood vessels with uniform or non-uniform cross-section by assuming the blood to be Non-Newtonian, incompressible and homogeneous fluid.

Shah and Siddiqui (2011) investigated the influence of peripheral layer viscosity on physiological characteristics of blood flow through stenosed artery using Power-law fluid model. They observed that the resistance to flow decreases as stenosis shape parameter increases and increases as stenosis length, stenosis size, peripheral layer viscosity increases. **Biswas and Laskar (2011)** investigated that axial velocity and flow rate increase with slip but decrease with yield stress and wall shear stress increases in Herschel-Bulkley fluid in comparison with corresponding Newtonian fluid. **Bali and Awasthi (2012)** studied the flow of blood through a

multistenosed artery under the influence of external applied magnetic field. The effect of non-Newtonian nature of blood in small blood vessels has been taken into account by modeling blood as a Casson fluid. The effect of magnetic field, height of stenosis, parameter determining the shape of the stenosis on velocity field, volumetric flow rate in stenotic region and wall shear stress at surface of stenosis are obtained and shown graphically. **Yadav and Kumar (2012)** studied the non-Newtonian behavior on blood flow through stenosed artery with power law fluid model. They gave the result that the resistance-to flow increases with stenosis size for different value of flow index behaviour.

Singh and Singh (2013) studied the blood flow through time-dependent artery with mild stenosis. They have shown effect of time on resistance to flow, volumetric flow rate, axial velocity and shear stress analytically and graphically. Expressions for dimensionless discharge variable and dimensionless shear stress variable are also obtained by them. Critical value of Reynolds number at which separation occurs has been found by them.

The flow characteristics through specific concentric and eccentric plaque formations are investigated by **Gurleen (2013)** via Large Eddy Simulation (LES) turbulence technique considering pulsatile flow conditions adjusted for a single frequency-sinusoidal motion (SIN) and for the coronary arteries namely the Left Anterior Descending (LAD) and Right Coronary Artery (RCA). **Ha and Lee (2014)** investigated the effects of pulsatile swirling inlet flows with various swirling intensities on the flow field in a stenosis model are experimentally using a particle image velocimetry velocity field measurement technique. **Akbar (2014)** discussed the blood flow analysis of Prandtl fluid model in tapered stenosed arteries. The governing equations for considered model are presented in cylindrical coordinates. Perturbation solutions are constructed for the velocity, impedance resistance, wall shear stress and shearing stress at the

stenosis throat. He observed that due to increase in Prandtl fluid parameters, the stenosis shape and maximum height of the stenosis the velocity profile decreases.

Akbar and Butt (2015) observed that the velocity profile is symmetric for all the parameters and when we increase slip parameter α then there will be more resistance between blood and arteries, hence the blood flow slows down and velocity profile decreases. They also noticed that velocity is high for all the parameters in case of pure water as compare to Cu-water because copper makes arteries more flexible that makes the blood flow speed slow. **Bhatnagar et al. (2015)** study the effect of slip velocity and shape of stenosis on non-Newtonian flow of blood through a stenosed arterial segment. The influence of stenosis shape parameter, slip velocity, stenosis height and yield stress on blood flow through a stenosed artery has been examined by them. **Akbar (2016)** studied the blood flow of a Walter's B fluid through a tapered artery with a stenosis modeled in cylindrical coordinates system. The expressions for velocity, resistance impedance, wall shear stress and shearing stress at the stenosis throat have been evaluated by him. **Nadeem and Najaj (2016)** investigated that the resistance impedance to blood flow decreases due to the increase in the concentration of nanoparticle volume fraction and also reveals that the silver nanoparticles are more helpful to reduce the resistance impedance to blood flow.

Zaman and Sajid (2017) studied pulsatile flow of blood through a porous-saturated stenotic artery under the influence of periodic body acceleration. The constitutive equation of cross model is considered to characterize the blood and also modified form of Darcy's law applicable to Cross model is used in their study. The shape of the stenosis in the arterial lumen is chosen to be overlapping w-shape. The velocity, flow rate and shear stress increase while resistance to flow decreases with greater permeability parameter has been obtained by them.

Majee and Shit (2017) investigated a numerical investigation of unsteady flow of blood and heat transfer has been performed with an aim to provide better understanding of blood flow through arteries under stenotic condition. The blood is treated as Newtonian fluid and the arterial wall is considered to be rigid having deposition of plaque in its lumen. They reveal that the increase in the magnetic field strength up to 8 T, does not causes any damage to the arterial wall, but the study is significant for assessing temperature rise during hyperthermic treatment.

(c) Broad out lines of the work:

The present work of research has been planned to be workout in to the following chapter:

- (i) Introduction
- (ii) Review of the Literature
- (iii) Problems on Blood flow obeying Herschel-Bulkley fluid, Casson fluid and Power law fluid through an artery with radially non-symmetric mild stenosis
- (iv) Problems on effect of flow indices on blood resistance flow in an atherosclerotic artery by abnormal segments
- (v) Problems on effect of Hematocrit on wall shear stress for blood flow through tapered artery
- (vi) Problems on MHD flow of blood through overlapping arteries, composite and time dependent geometry of arteries.
- (vii) Problems on Pulsatile flow of blood under stenotic condition
- (viii) Problems on peristaltic flow of blood through artery
- (ix) Problems of arterial blood flow with external effect such as body acceleration and porous effect.

The relevant references will be given at the end of thesis.

The nomenclature of aforesaid chapters is tentative. It may be altered in accordance our investigation.

(d) Preliminary work done on the line:

Before preparing the present synopsis, I have read a number of textbooks on Biomathematics. I have also gone through the current literature regarding my research project. Here is the list of the textbooks consulted by me during this period.

1. Computational fluid dynamics: A.J. Chorin
2. Numerical Methods for scientific and Engineering computation: M.K. Jain, S.R.K. Iyenger and R.K. Jain
3. Mathematical Biology: Burton
4. Mathematical Models in Biology and Medicine: J.N. Kapur
5. Mathematics in medicine and life science: Peskin and Hoppensteadt
6. Biodynamics circulation: Y.C. Fung
7. Computational cell biology: Christopher P. Fall, Eric S. Marland, John M. Wanger and John J. Tyson

I have been also in constant touch with the following Indian and Foreign Journals:

1. Journal of Biomechanics
2. In Silico Biology
3. Indian Journal of Biotechnology
4. Computational Fluid Dynamics

5. Journal of Applied Mathematics
6. Indian Journal of Biochemistry and Biophysics
7. Journal of Mathematical Biology
8. Bulletin of Mathematical Biology
9. Bulletin of Mathematical Biology
10. SIAM Journals
11. IEEE Journals
12. Journal of Theoretical Biology
13. Journal of Mathematical Bioscience
14. IMA Journal of Mathematical Application in Medicine and Biology

(e) **Proposed research design, tools, methodology, hypothesis and tentative conclusions:**

To analyze satisfactorily blood flow in the human system, we have to know the basic principles of fluid dynamics such as equation of continuity and momentum for fluid flow in different system of coordinate, solution of Hagen-Poiseuille flow in a circular tube as well as special characteristics of cardiovascular system. Since blood flow is mostly inlet length flow therefore we have to know inlet length flows i.e. flows in the entrance regions of pipes before these flows become fully established. To distinguish between laminar and turbulent flows, we have to introduce the concept of Reynold number of the flow. Since the relation between stress and strain rate is not linear in blood, we also introduced the various types of non-Newtonian fluids exhibiting this relationship.

The special characteristics of flow of blood in some arteries in pulsatile flow. We will use the flow when the external pressure gradient is a periodic function of time, and extend this study to pulsatile flows in elastic tubes. This is useful since blood vessel walls are elastic.

The research methodology undertaken to solve the problems are problem formulation, non-dimensionalization of the problem, radial coordinate transformation and numerical computation.

Our proposed research programme will include certain new problems which will meet the need and interest of present day. We shall make use of methods established by prominent mathematicians to formulate the problems and find out the solutions. During the research work we shall use the tools on special functions, higher order differential equation, integral transform and numerical method of integration. The method of dimensional analysis may be used for obtaining the solution of differential equations under certain boundary condition. To draw the conclusion, the method of plotting the graphs among different parameters as well as interpretation for obtained results, may be of great use.

Recent computational techniques would use to solve the basic equation of the problems. Proper programme will be prepared and existing programming will also be applicable in some important case.

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