SYNOPSIS

THE PERFORMANCE PREDICTION OF MACHINING SYSTEMS USING QUEUE THEORETIC APPROACH

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BY

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PURPOSE OF STUDY

The objective of present study is based on the philosophy that interaction should evolve around industrial problems which mathematicians may be able to solve in ‘real time’. In this way both industry and mathematics will be benefited; industry by increase of mathematical knowledge and ideas brought to bear upon their concerns and mathematics through the infusion of exciting new problems. There is a substantial number of industrial problems to which tools of Operations Research are applicable. New techniques based on operational research have become popular as new technology comes on line.

With a remarkable advancement in science and technology, machines have added to almost every field in different frame works. The machines have become an integral part of human civilization, as the life has become comfortable due to industrial development. Our total dependence on machines can be attributed to the fact that the machining systems today have entered into every sphere of human activities. Recent times have witnessed the emergence of high tech machining systems required in today’s world of competitions in the field of manufacturing, production, telecom and IT sectors, etc.. Queueing analysis of machining systems helps in evaluating the measures of effectiveness in such systems, which are concerned with services or manufacturing.

The machines are unreliable and thus always prone to failures. These failed machines are queued up for repair under the supervision of caretaker which also experience the waiting phenomenon. Thus, the problem falls in the category of queueing theory. The machine failures obviously affect the organization in terms of production, money, goodwill in the market etc. For the smooth functioning of the machining system and to compensate
the loss due to machine breakdowns, the organizations have to keep spares along with the arrangement of repair facility. Now the role of the organization’s system designer comes into the picture whose task is to suggest an optimal combination of spares and repairmen, which the organization should employ so that the expenses of keeping these facilities should not affect the system’s profit too much.

For this purpose, the mathematical modeling (based on queueing theory) of machine repair problem plays an important role which helps to analyze and predict the performance of the machining system quantitatively. This gives an opportunity to system designers to reduce the machine interference due to failure of units of machining system to a reasonable extent. Therefore, foreseeing a wide scope to work in this important and interesting field, it motivates us to innovate and work in this particular field.

Our motivation to study the machining systems comes from the serious problems being faced by the industries due to machine failures and the resulting slow progress in technological development throughout the world. Industries are making efforts to cope up with the increasing complexity in the machining systems, fast changing technology and varying market patterns. Technological development and requirement of modern society are competing with each other. Therefore, with the development of modern technology and its role in industrial development, it is quite important to keep a pace with recent researches and developments in order to quantify the performance and effectiveness of various systems such as computer and communication system, manufacturing/production system, etc. Now-a-days, it has been widely applied in various industrial service systems including the maintenance systems, repairing of the machines, batch processing, inspection stations, computer facilities etc. It has also been applicable in social service systems such
as various health care systems, public transportation systems and reticulation of mass distribution systems.

The aim of present study is to analyze the performance of machining systems involved in some manufacturing/production or service systems. We shall attempt to examine and provide solutions to the problems being faced due to machine failures. Classical queueing techniques as well as numerical techniques will be used to analyze the queueing models used for various problems of machining systems. The performance measures in each case will be established, which may be helpful to the system designers and decision makers to plan a strategy for the smooth functioning of the system. The optimal control policy for some machine interference problems will be suggested so as to ensure smooth running of the system at optimum cost under techno-economic constraints.

We hope that our study would not only help in predicting the performance and improving the performability of the existing systems, but also support the system engineers in creating optimum designs of the concerned architecture of machining system. Our work will be at par with other Universities/Institutions in the country and elsewhere. The models analyzed under proposed study will also be of applied nature and will attract the attention of mathematicians and operations research scientists. The study will be useful as decision-making tools to achieve maximum output subject to resource constraints.

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[A] Title: PERFORMANCE ANALYSIS OF MACHINING SYSTEMS USING QUEUE THEORETIC APPROACH

[B] PRESENT STATE OF KNOWLEDGE

With the fast growing technology, the industries are struggling to muddle through the mounting complexity in the machining systems due to computerization, automated manufacturing flexibility in demand, fast change in technology, market patterns, etc. The technological advancement and call for modern society are contesting against each other. Therefore, with the growth of modern society and its role in industrial development, it is indispensable to be in touch with recent research and development, which can provide a right direction to design highly sophisticated machining systems.

Queueing theory has emerged as a powerful mathematical tool to analyze the machining systems in different frameworks because of its ability to simulate most real time systems. Machine repair problems are applicable in all branches of engineering, manufacturing system/production processes, information system, transportation system, computer and communication networks, etc.

The machining systems considered in the present study provide some tactical ideas concerning the concept of maintenance and replacement policy via repair facility and standby support. Here we present a survey concerning an appraisal of machine repair problems (MRPs) highlighting the historical advancement of queueing models. The survey crops up historically, beginning with the developments in 1985 when the first published review on machine interference models by Stecke and Aronson (1985) came into existence. Machine interference problems have been studied by many queue theorists for a variety of congestion situation. They have used different assumptions and approaches for
this purpose. The early notable contribution in this area were due to Palm (1947), Benson and Cox (1951), Naor (1956), Ferdinand (1971), Albright (1980), Karmeshu (1990) and many more.


The appearance of queues or waiting lines is a universal phenomenon, thus making the queueing theory applicable in almost every walk of life; the area of repairable machining system is particularly suited to such an application. The machines along with the spares only cannot fully serve the purpose of increasing waiting lines. To reduce the exceeding workload, the incorporation of additional repairmen along with the permanent repair facility is suggested. The research work including this idea has also been carried out by many researchers. Knessl (1991) observed the transient behaviour of repairman

Machine repair model with standbys, discouragement with vacationing unreliable repairmen was developed by Singh and Maheshwari (2013).

The general problem of reliability of machine repair problem holds a very wide range of problems intended to ensure and maintain a high reliability both of individual elements and the entire system as a whole. The problem has constantly attended the engineering and technical progress and has always found reasonable solution at the level of technical possibilities. Various mathematicians have worked in the field of reliability analysis of machining systems. Availability and reliability of an n-unit warm standby redundant system was examined by Kalpakam and Hameed (1984). Availability and mean life time prediction of multistage degraded system with partial repairs was measured by Pham et al. (1997). Jain et al. (2004) studied reliability analysis of redundant repairable system with degraded failure. An artificial neural network for modeling reliability, availability and maintainability of a repairable system was proposed by Rajpal et al. (2006). Optimal design of a maintainable cold standby system was specified by Haiyang et al. (2007). Y and Meng (2011) investigated reliability analysis of a warm
standby repairable system with priority in use. Reliability analysis of machining systems by considering system cost is investigated by Zhang and Liling (2015). Zhou et al. (2017) developed a service facility maintenance model with energy-delay tradeoff. The maintenance model integrates individual control chart with queueing approach.

[C] BROAD OUTLINES OF THE WORK

Our investigation is particularly focused on the analysis of machine repair problems in different frameworks. For the modeling of the problems and performance prediction, the techniques to be incorporated will be either classical queueing techniques or numerical techniques. The chapter wise organization of the thesis as follows:

(i) General Introduction
(ii) Review of Literature
(iii) Machine repair problem with spares and state dependent rates
(iv) Machine repair problem with spares and N-Policy vacations
(v) Machine repair problem with spares and additional repairmen
(vi) Machine repair system by generating function technique
(vii) Multi-repairmen machine repair problem with M operating units and k type of warm spares
(viii) Machine repair problem in production system with spares and server vacations

The relevant references will be given at the end of thesis.

The nomenclature of aforesaid chapters is tentative. It may be altered in accordance our investigation.
Mathematical Formulation:

A basic queuing system is formed from three general elements.

(i) The arrival process of users in the system;

(ii) The order in which users obtain access to the service facility, one they join the queue.

(iii) The service process and departure from the system.

** Arrival** refers to the average number of customers who require service within a specific period of time.

** Customers** can be people, work-in-process inventory, raw materials, incoming digital messages, or any other entities that can be modeled who are to wait for some process to take place it may be infinite or finite also is said size of queue.

** A server** can be human worker, a machine, or any other entity that can be as executing some process for waiting customers.

While analyzing a queuing system we can identify some basic components of it. Namely, **Queue Discipline** refers to the priority system by which the next customer to receive service is selected from a set of waiting customers. One common queue discipline is first-in-first-out, or FIFO. Scheduling adopt by server refer, how to work server. Server follows the following discipline. The server in accepting customers for service. In this context, the rules as “first-come-first-served” (FCFS), “last-come-last-served” (LCLS), and “random selection for service” (RS) are self explanatory. Others such as “round robin” and “shortest processing time” may need some elaboration. In many situations customers in service classes get priority in service over others. There are many other queue disciplines which have been introduced for sufficient operation of computers and communication systems.
Also, there are other factors of customer behavior such as balking, reneging, and jockeying that require consideration as well.

**Service rate** (or service capacity) refers to the overall average number of customers a system can handle in a given time period.

**Utilization** refers to the proportion of time that a server (or system of servers) is busy handling customers.

**Stochastic Processes** are systems of events in which the times between events are random variable. In queuing models, the patterns of customer arrivals and service are modeled as stochastic process based on probability distribution.

**Kendal (1953)** classified the queues and according to this description following symbols are used in queuing problems.

- * M: Exponential
- * D: Deterministic (constant)
- * $H_k$: Hyper exponential with parameter $k$ (mixture of $k$ exponentials)
- * $E_k$: Erlang with parameter $k$
- * G: General (all)

A conference on standardization of notation in queuing theory agreed in 1971 to extend Kendall’s notation. It is as A/B/m/K/N/Z.

where,

* A indicates the distribution of interarrival times,
* B denotes the distribution of the service times,
* m is the number of servers,
* K is the capacity of the system, that is the maximum number of customers staying at the facility (sometimes in the queue),

* N denotes the number of sources,

* Z refers the service discipline

The some well known basic formulas of queuing theory is as follows:

**Erlang’s Formulas:** Erlang’s task can be formulated as, what fraction of incoming calls is lost because of the busy line at the telephone exchange office. Of course, the answer is not so simple, since we first should know the inter-arrival and service time distributions. After collecting data Erlang verified that the Poisson-process arrival and exponentially distributed service appropriate mathematical assumptions. He considered the M/M/n/n and M/M/n cases, that the system where the arriving calls lost because all the servers are busy, and where the calls have to wait for service, respectively. Assuming that the arrival intensity is \( \lambda \) he derived the formulas for loss and delay systems, called Erlang B and C ones, respectively.

Denoting \( \rho = \lambda / \mu \), the steady state probability that an arriving calls is lost can be obtained in the following way

\[
P_n = \frac{\rho^n}{\sum_{k=0}^{n} \frac{\rho^k}{k!}} = B(n, \rho)
\]

where \( B(n, \rho) \) is well-known Erlang B-formula, or loss formula. It can be easily seen that the following recurrence relation is valid

\[
B(n, \rho) = \frac{\rho B(n-1, \rho)}{n + \rho B(m-1, \rho)}, \quad n = 2, 3, \ldots \]
\[ B(1, \rho) = \frac{\rho}{1 + \rho}. \]

Similarly, by using the b-formula the steady state probability that an arriving customer has to wait can be written as

\[ C(n, \rho) = \frac{n B(n, \rho)}{n - \rho(1 - B(n, \rho))}, \]

which is called Erlang C-formula, or Erlang’s delay formula? It should be mentioned that b-formula is insensitive to the service time distribution, in other words it remains valid for any service time distribution with mean \( \frac{1}{\mu} \).

**Little’s Law:** It is also known as Little’s result, or Little’s theorem, is perhaps the most widely used formula in queuing theory was published by j. little in 1961. It is simple to state and intuitive, widely applicable, and depends only on weak assumptions about the properties of the queuing system.

A conservation law that applies to the following general setting:

\[ \text{Input} \rightarrow \text{System} \rightarrow \text{Output} \]

**Input:** Continuous flow or discrete units.

**System:** Boundary is all that is required.

**Output:** Same as input, call throughput.

Two possible scenarios:

(iv) System during a “cycle” (empty \( \rightarrow \) empty, finite horizon);

(v) System in steady state/in the long run (for example, over many cycles).

Quantities that are related via Little’s law (long-run averages, or time-averages):

(vi) \( \lambda = \) rate at which units arrive (=long-run average rate at which units depart) = throughput-rate, whose units are \( \text{quantity/time-unit, or \#/time-unit} \);
(vii) \( L \) = inventory/quantity/number in the system
(e.g., WIP: Work-In-Process, Customers)

(viii) \( W \) = time a unit spends in the system = throughput time

Thus, the Little’s Law says that, under steady state conditions, the average number of items in a queuing system (\( L \)) equals the average rate (\( \lambda \)) at which items arrive multiplied by the average time (\( W \)) that an item spends in the system, and algebraically expressed as:

\[
L = \frac{\lambda}{\mu}W
\]

M/M/1 model

In this model the arrival times and service rates follow Markovian distribution or exponential distribution which are probabilistic distributions, so this is an example of stochastic process. In this model there is only one server. The important results of this model are:

1. Average number of customers in the system \( L = \frac{\rho}{1-\rho} \)

2. Average number of customers in the system \( L_q = \frac{\rho^2}{1-\rho} \)

3. Expected waiting time in the system \( W = \frac{L}{\lambda} = (1/\lambda) \frac{\lambda}{\mu-\lambda} = \frac{1}{\mu-\lambda}. \)

4. Expected waiting time in the queue \( W_q = \frac{L_q}{\lambda} = (1/\lambda) \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\lambda}{\mu(\mu-\lambda)}. \)
In this section we investigate the following question: when we look at the number of customers in a queuing system at “random points” in time, do then have all “random points” the same properties or do there exist points where the results differ fundamentally?

The answer is: yes, it makes a difference how you choose the points. A simple example illustrating this is the D/D/1 queue (with fixed interarrival times of $\lambda$ seconds and fixed service times of $\mu$ seconds, $\lambda < \mu$. If we now choose “randomly” the arrival times of new customers as our random points it is clear that we will see zero customers in the system every time (since customers are served faster than they arrive). If we choose the random times with uniformly distributed time differences there is always a probability of $\lambda/\mu > 0$ to find a customer in the system. Thus, the final result of this section is that arrival times are not random times. One important exception from this rule is when the arrival times come from a Poisson process (with exponentially distributed interarrival times). In this case the arrival times are “random enough” and a customer arriving to a queue in the steady state sees exactly the same statistics of the number of customers in the system as for “real random times”. In a more condensed form this is expressed as Poisson Arrivals See Time Averages, abbreviated with PASTA. This property comes also from the memoryless property of the exponential distribution.

[D] PRELIMINARY WORK DONE ON THE LINE

I have gone through the literature and consulted some reference books and textbooks related to my research topic. Some worth mentioning books consulted are as follows:
Operations Research: Taha, H.M.
Introduction of Queueing Theory: Cooper, R.B.
Fundamentals of Queueing Theory: Gross, D. and Harris, C.M.
Queueing Systems, Vol. 1 & 2: Klienrock, L.
Stochastic Processes: Medhi, J.
Probability & Statistics with Reliability, Queueing & Computer Science Applications: Trivedi, K.S.
Markovian Queues: Sharma, O.P.
Reliability of Engineering System - Principles and Analysis: Ryabinin, I.
Performance Analysis of Manufacturing Systems: Altiok, T
Queueing Networks with Blocking- Exact and Approximate Solutions: Perros, H.G.

I am in constant touch with the following research journals:

Opsearch
Operations Research Letters
European Journal of Operations Research
Applied Mathematical Modeling
International Journal of Management & Systems
International Journal of Pure and Applied Mathematics
Journal of Operations Research Society
Journal of Indian Statistical Association
These days the internet facility is quite beneficial for the researchers to keep a pace with latest updates regarding the developments in their respective research fields. It has now become an easy task to search research articles, their abstracts, references and other necessary information. The most common websites consulted are www.sciencedirect.com, www.inderscience.com and www.springerlink.com.

After surveying the literature, I am working on the following research problems.

- Queueing system with second optional service.
- Machine repair problem with mixed spares, discouragement and additional repairmen.
- The transient analysis of Markovian machine repair model with balking and reneging by using birth-death process.

Proposed research design, tools, methodology, hypothesis and tentative conclusions:

The hypothesis of the study will be based on optimization. The determination of highest or lowest value over some given range is called optimization. A queuing model can be maximized to profit or minimized for loss. The word optimization is used as general term in either case. Cost and optimization can be classified into three categories:
(i) Cost and optimization engineering: It deals with the topics of engineering economy.

(ii) Operations research: It is mathematical oriented and is concerned with advance techniques for solving complicated optimization problems using mathematical models.

(iii) Management science: It is relatively new category originating from operations research and is aimed at overall policy and decision-making.

(iv) The queuing models have been classified into two general types

Though queuing theory provides us a scientific method of understanding the queues and solving such problems, the theory has certain limitations which must be understood while using the technique and in conjunction with the other decision analysis methods like simulation and regression. Most of the limitations are the basic assumptions for application of queuing models. Some of the limitations of the queuing models are enumerated below:

(i) Based on assumption that service time is known.

(ii) Assumes steady state.

(iii) Service times are independent from one another.

(iv) Service rate is known.

(v) Service rate is greater than arrival rate.

(vi) Service time is described by negative exponential probability distribution.

(vii) The waiting space for the customer is usually limited.

(viii) The arrival rate may be state dependent.
(ix) The arrival process may not be stationary.

(x) Services may not be rendered continuously.

(xi) The queuing system may not have reached the steady state. It may be, instead, in transient state.

(xii) Mathematical distributions, which we assume while solving queuing theory problems, are only a close approximation of the behavior of customers, time between their arrival and service time required by each customer.

(xii) Most of the real life queuing problems are complex situation and very difficult to use queuing theory technique, even then uncertainty will remain.

(xiii) Many situations in industry and service are multi-channel queuing problems. When a customer has been attended to and the service provided, it may still have to get some other service from another service and may have to fall in queue once again. Here the departure of one channel queue becomes the arrival of the other channel queue. In such situations the problem becomes still more difficult to analyze.

Queuing model may not be the ideal method to solve certain very difficult and complex problems and one may have to resort to other techniques like Monte-Carlo simulation method.

Some approaches used for the solution of mathematical models under consideration are as follows:

**Analytical techniques:**

- Markovian birth death process
- Recursive technique
- Product type method
- Supplementary variable technique
- Generating Function Method

**Numerical techniques:**

- Runge-Kutta technique
- Successive Over Relaxation method (SOR)
- Matrix Method, Matrix geometric method
- Neuro-Fuzzy approach

To plan a fruitful strategy for the smooth functioning of the system, some performance measures for the system under study will be obtained by using suitable techniques which may be helpful to the system designer and decision makers. Worth mentioning are the following:

- Reliability/Availability
- State probabilities.
- Queue size distribution.
- Expected number of spares
- Expected number of repairmen
- Expected number of failed machines in the queue/system
- Cost function
- Numerical illustration to validate the analytical results.
To draw the conclusion, the method of plotting the graphs among different parameters as well as interpretation for obtained results, may be of great use.

Recent computational techniques would use to solve the basic equation of the problems. Proper programme will be prepared and existing programming will also be applicable in some important case.
REFERENCES


